Optical Communication Systems: Problem Set 2 (Optical Fibers)

P2-1 A multimode step index fiber with a 50-μm core diameter is designed to limit the intermodal dispersion to 10 ns/km. Calculate the refractive index of the core. What is the numerical aperture of this fiber? What is the limiting bitrate for transmission over 10 km at 0.88 μm. Use 1.45 for the refractive index of the cladding.

P2-2 (a) It is generally accepted that when the spread ΔT of the light pulse becomes wider than 70% of the bit period $T_B = 1/B$, information transmission becomes unreliable. Now consider a multimode SIF which gives a total pulse broadening of 95 ns over a 5 km length. According to above rule i.e. $BΔT<0.7$, estimate the bandwidth–length ($Δf \cdot L$) product for the fiber using the NRZ code.

(b) Another more accurate estimate of the maximum bit rate for an optical channel with dispersion may be obtained by $5BΔT \leq 1$. A single-mode step index fiber has a bandwidth–length product of 10 GHz-km. Estimate the RMS pulse broadening over a 40 km digital optical link without repeaters consisting of the fiber using the RZ code.

Hint: For the NRZ code $Δf=B/2$ and for the RZ code $Δf=B$.

P2-3 A single-mode fiber has an index step $n_1-n_2=0.005$. Calculate the core radius if the fiber has a cutoff wave length of 1 μm. Estimate the spot size (FWHM) of the fiber mode and the fraction of the mode power inside the core when this fiber is used at 1.3 μm. Also, calculate the normalized frequency. Use $Δ=3.45 \times 10^{-3}$.

P2-4 A single-mode step fiber index has a core diameter of 7 μm and a core refractive index of 1.49. Estimate the shortest wavelength of light which allows single-mode operation when the relative refractive index difference for the fiber is 1%. Also, it is required to increase the fiber core diameter to 10 μm while maintaining single-mode operation at the same wavelength. Estimate the maximum possible relative refractive index difference for the fiber.

P2-5 A graded index fiber with a core axis refractive index of 1.5 has a characteristic index profile (α) of 1.90, a relative refractive index difference of 1.3% and a core diameter of 40 μm.

(a) Estimate the number of guided modes propagating in the fiber when the transmitted light has a wavelength of 1.55μm.

(b) Determine the cutoff value of the normalized frequency for single-mode transmission in the fiber.

P2-6 A multimode, optimum, near-parabolic profile graded index fiber has a material dispersion parameter of $D_M=30$ ps/(km-nm) when used with a good LED source of RMS spectral width $σ_2=25$ nm. The fiber has a numerical aperture of 0.4 and a core axis refractive index of 1.48. Estimate the total RMS pulse broadening (i.e. due to both the intermodal dispersion and the GVD) per kilometer within the fiber assuming the waveguide dispersion to be negligible. Hence, estimate the BL product for the fiber.

P2-7 An approximation for the normalized propagation constant in a single-mode step index fiber is:

$$b(V) \approx (1.1428 - 0.996 / V)^2$$

(a) Obtain a corresponding approximation for the waveguide parameter $Vd^2(Vb)/dV^2$ and hence write down an expression for the waveguide dispersion in the fiber.

(b) Estimate the waveguide dispersion in a single-mode step index fiber at a wavelength of 1.34 μm when the fiber core radius and refractive index are 4.4 μm and 1.48 respectively.
(c) A single-mode step index fiber exhibits material dispersion of 7 ps/(km-nm) at an operating wavelength of 1.55 μm. Using the approximation in pat a, estimate the fiber core radius which will enable the waveguide dispersion to cancel the material dispersion so that zero intramodal dispersion is obtained at this wavelength. The refractive index of the fiber core is 1.45.

P2-8 (a) Estimate the limiting bit rate for a 60-km single mode fiber link at λ₁=1.3- and λ₂=1.55-μm wavelengths assuming transform-limited, 50-ps (FWHM) input pulses. Assume that β₂ =0 and -20 ps ²/km and β₃ = 0.1 ps ³/km and 0 at 1.3- and 1.55-μm wavelengths, respectively. Solve the problem for (i) a light source with negligible spectral width and (ii) a light source of RMS spectral width 25 nm. 
(b) By neglecting the spectral width, redo part a for a chirped Gaussian pulse for which C=-6.

P2-9 A dual mode fiber operating at a wavelength of 1.55 μm has a core refractive index of 1.46, a relative refractive index of 2%, and a core radius of 2.1 μm.
(a) Determine the values of the waveguide dispersion parameter Dₓ using Figure P2-9, the material dispersion parameter Dₓ, and the dispersion parameter D in terms of ps/(km-nm). Use Figure 2.9 of the textbook to determine the material dispersion.
(b) After 100 km of transmission through a step-index single-mode fiber, the dual-mode fiber is used as a dispersion compensator. What is the required length of the dual-mode fiber?

![Figure P2-9](image)

**Fig. P2-9** \( Vd^2(Vb)/dV^2 \) of \( \text{LP}_{11} \) (\( HE_{21} \), \( TE_{01} \), \( TM_{01} \)) versus \( V \) for calculating the dispersion parameter.

P2-10 A multimode step index fiber has a relative refractive index difference of 1% and a core refractive index of 1.46. The maximum bitrate that may be obtained with a particular source on a 5 km link is 3.5 Mb/s.
(a) Determine the RMS pulse broadening per kilometer resulting from chromatic dispersion (GVD) mechanisms.
(b) Assuming waveguide dispersion may be ignored, estimate the RMS spectral width of the source used, if the material dispersion parameter for the fiber at the operating wavelength is 90 ps/(km-nm).

P2-11 The characteristics of polarization maintain fibers (PMFs) are described not only by the modal birefringence or beat length but also by the mode coupling parameters or polarization crosstalk as well as their transmission losses. The mode coupling parameter or coefficient \( h \), which characterizes
the PM ability of fibers based on random mode coupling, proves useful in the comparison of different lengths of PMF. It is related to the polarization crosstalk (CT) by:

\[ CT = 10 \log_{10} \left( \frac{P_2}{P_x} \right) = 10 \log_{10} \tanh(hL) \]

where \( P_x \) and \( P_2 \) represent the optical power in the excited (i.e. unwanted) mode and the coupled (i.e. launch) mode, respectively, in an ensemble of fiber length \( L \). This equation applies with greater accuracy to two-polarization fibers because the crosstalk in a single-polarization fiber becomes almost constant around –30 dB and is independent of the fiber length beyond 200 m.

A 3.5 km length, of two-polarization mode PMF has a polarization crosstalk of –27 dB at its output end. Determine the mode coupling parameter for the fiber

**P2-12** The birefringent coherence is maintained over a length of fiber \( L_{hc} \) (i.e. coherence length) when:

\[ L_{hc} \approx \frac{c}{B_m \delta f} = \frac{\lambda^2}{B_m \delta \lambda} \]

where \( \delta \lambda \) is the source linewidth and \( \delta f \) is the uncorrelated source frequency width. The beat length in a single-mode optical fiber is 9 cm when light from an injection laser with a spectral linewidth of 1 nm and a peak wavelength of 0.9 μm is launched into it. Determine the modal birefringence and estimate the coherence length in this situation. In addition calculate the difference between the propagation constants for the two orthogonal modes.

**P2-13** (a) Calculate the threshold power for stimulated Brillouin scattering for a 50-km fiber link operating at 1.3 μm and having a loss of 0.5 dB/km. How much does the threshold power change if the operating wavelength is changed to 1.55 μm, where the fiber loss is only 0.2 dB/km? Assume that \( A_{eff}=50 \mu m^2 \) and \( g_s=5 \times 10^{-11} \) m/W at both wavelengths.

(b) Determine the threshold optical power for SRS within the fiber at this wavelength assuming \( g_s=5 \times 10^{-13} \) m/W.

**P2-14** Calculate the power launched into a 40-km-long single-mode fiber for which the SPM-induced nonlinear phase shift becomes 180°. Assume \( \lambda=1.55 \) μm, \( A_{eff}=40 \) μm², \( \alpha=0.2 \) dB/km, and \( n_2=2.6 \times 10^{-20} \) m²/W.

**Answers:**

(2-1) \( n_l=1.453, \) NA=0.092, \( B_{max}=10 \) Mb/s

(2-2) (a) \( \Delta f \cdot L=18.48 \) MHz-km; (b) \( \sigma=800 \) ps

(2-3) \( a=3.18 \) μm, \( w=4.34 \) μm, \( \Gamma=0.66 \)

(2-4) \( \lambda=1.93 \) μm, \( \Delta=0.005 \)

(2-5) (a) \( V=19.61 \rightarrow N=94 \); (b) \( V=3.45 \)

(2-6) \( \sigma=774 \) ps, \( BL=323 \) Mb/s-km

(2-7) (a) \( Vd^2(Vb)/dV=1.984V^2 \);

(b) \( D_w = -8.37 \times 10^{-11} \lambda/(n_1a^2)=-3.92 \)

ps/(nm-km); (c) \( a=3.6 \) μm

(2-8) (a) (i) \( B_1=11.8 \) Gb/s; \( B_2=7.07 \) Gb/s; (ii) \( B_1=75.88 \) Mb/s, \( B_2=10.62 \) Mb/s; (b) \( B_1=75.81 \) Mb/s, \( B_2=10.62 \) Mb/s

(2-9) \( V=2.49, D_w=-314, D_M=20, L=5.1 \) km

(2-10) (a) 2.6 ns/km; (b) \( \sigma_s=28.8 \) nm

(2-11) \( h=5.7 \times 10^{-7} /m \)

(2-12) \( B_n=10^{-5}, L_{hc}=81 \) m, \( \Delta \beta=69.8 \)

(2-13) (a) \( P_{th1}=2.42 \) mW, \( P_{th2}=1 \) mW; (b) \( P_{th1}=184.54 \) mW, \( P_{th2}=81.8 \) mW

(2-14) \( \gamma=0.0026, P_{in}=61 \) mW