1. The $x$ and $y$ components of velocity for a two-dimensional flow are $u = 3.0 \text{ ft/s}$ and $v = 9.0x^2 \text{ ft/s}$ where $x$ is in feet. Determine the equation for the streamlines and graph representative streamlines in the upper half plane.

2. In addition to the customary horizontal velocity components of the air in the atmosphere (the “wind”), there often are vertical air currents (thermals) caused by buoyant effects due to uneven heating of the air as indicated in below Figure. Assume that the velocity field in a certain region is approximated by $u = u_0, u = v_0 (1 - y/h)$ for $0 < y < h$ and $u = u_0, u = v_0$ for $y > h$. Plot the shape of the streamline that passes through the origin for values of $u_0/v_0 = 0.5, 1.0, \text{ and } 2.0$.

3. As a valve is opened, water flows through the diffuser shown in below Figure at an increasing flow rate so that the velocity along the centerline is given by $V = u \hat{i} = V_0 (1 - e^{-ct}) (1 - x/l) \hat{i}$ where $u_0, c$ and $l$ are constants. Determine the ac acceleration as a function of $x$ and $t$. If $V_0 = 10.0 \text{ ft/s}$ and $l = 5.0 \text{ ft}$ what value of $c$ (other than $c = 0$) is needed to make the acceleration zero for any $x$ at $t = 1.0 \text{ s}$. Explain how the acceleration can be zero if the flow rate is increasing with time.

4. A nozzle is designed to accelerate the fluid from $V_1$ to $V_2$ in a linear fashion. That is, $V = ax + b$ where $a$ and $b$ are constants. If the flow is constant with $V_1 = 10.0 \text{ m/s}$ at $x_1 = 0.0 \text{ m}$ and $V_2 = 25.0 \text{ m/s}$ at $x_2 = 1.0 \text{ m}$ determine the local acceleration, the convective acceleration, and the acceleration of the fluid at points (1) and (2). The flow is not steady, but at the time when $V_1 = 10.0 \text{ m/s}$ and $V_2 = 25.0 \text{ m/s}$, it is known that $\partial V_1/\partial t = 20.0 \text{ m/s}$ and $\partial V_2/\partial t = 60.0 \text{ m/s}$.

5. For flow past a sphere as shown in below Figure, plot a graph of the stream-wise acceleration, $a_s$ the normal acceleration, $a_n$ and the magnitude of the acceleration as a function of $\theta$ for $0 \leq \theta \leq 90$ with $V_0 = 50.0 \text{ ft/s}$ and $a = 0.1, 1.0 \text{ and } 10.0 \text{ ft}$. At what point is the acceleration a maximum; a minimum?

* These exercises have been selected from several books.

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6. Air flows from a pipe into the region between a circular disk and a cone as shown in below Figure. The fluid velocity in the gap between the disk and the cone is closely approximated by $V = \frac{V_0 R^2}{r^2}$ where $R$ is the radius of the disk, $r$ is the radial coordinate, and $V_0$ is the fluid velocity at the edge of the disk. Determine the acceleration for and $r = 0.5$ and 2 ft if $V_0 = 5.0 \text{ ft/s}$ and $R = 2.0 \text{ ft/s}$.

7. Determine the magnitude and direction of the $x$ and $y$ components of the anchoring force required to hold in place the horizontal $180^\circ$ elbow and nozzle combination shown in below Figure. Also determine the magnitude and direction of the $x$ and $y$ components of the reaction force exerted by the elbow and nozzle on the flowing water.

8. A sheet of water of uniform thickness ($h = 0.01 \text{ m}$) flows from the device shown in below Figure. The water enters vertically through the inlet pipe and exits horizontally with a speed that varies linearly from 0 to 10 m/s along the 0.2-m length of the slit. Determine the $y$ component of anchoring force necessary to hold this device stationary.
9. Water flows vertically upward in a circular cross-sectional pipe as shown in Figure. At section (1), the velocity profile over the cross-sectional area is uniform. At section (2), the velocity profile is: \( V = w_c \left( \frac{R-r}{R} \right)^{1/7} \hat{R} \), where \( V = \) local velocity vector, \( w_c = \) centerline velocity in the axial direction, \( R = \) pipe radius, and \( r = \) radius from pipe axis. Develop an expression for the fluid pressure drop that occurs between sections (1) and (2).

10. Water flows from a large tank into a dish as shown in Figure. (a) If at the instant shown the tank and the water in it weigh \( W_1 \) lb, what is the tension \( T_1 \), in the cable supporting the tank? (b) If at the instant shown the dish and the water in it weigh \( W_2 \) lb, what is the force \( F_2 \), needed to support the dish?

11. Water is added to the tank shown in Figure through a vertical pipe to maintain a constant (water) level. The tank is placed on a horizontal plane which has a frictionless surface. Determine the horizontal force, \( F \), required to hold the tank stationary. Neglect all losses.
12. Five liters/s of water enter the rotor shown in below Figure along the axis of rotation. The cross-sectional area of each of the three nozzle exits normal to the relative velocity is $18.0 \text{ mm}^2$. How large is the resisting torque required to hold the rotor stationary? How fast will the rotor spin steadily if the resisting torque is reduced to zero and (a) $\theta = 0^\circ$, (b) $\theta = 30^\circ$, (c) $\theta = 60^\circ$?

13. Two water jets collide and form one homogeneous jet as shown in below Figure. (a) Determine the speed, $V$, and direction, $\theta$, of the combined jet. (b) Determine the loss for a fluid particle flowing from (1) to (3), from (2) to (3). Gravity is negligible.

14. The cone frustum in the figure contains incompressible liquid to depth $h$. A solid piston of diameter $d$ penetrates the surface at velocity $V$. Derive an analytic expression for the rate of rise $\frac{dh}{dt}$ of the liquid surface.

15. An incompressible fluid is squeezed between two disks by downward motion $V_o$ of the upper disk. Assuming 1-dimensional radial outflow, find the velocity $V(r)$.
16. Consider uniform flow past a cylinder with a V-shaped wake, as shown. Pressures at (1) and (2) are equal. Let $b$ be the width into the paper. Find a formula for the force $F$ on the cylinder due to the flow. Also compute $C_D = F/(\rho U^2 b)$.

17. A liquid jet of velocity $V_j$ and area $A_j$ strikes a single 180° bucket on a turbine wheel rotating at angular velocity $\Omega$. Find an expression for the power $P$ delivered. At what $\Omega$ is the power a maximum? How does the analysis differ if there are many buckets, so the jet continually strikes at least one?

18. Gravel is dumped from a hopper, at a rate of 650 N/s, onto a moving belt, as in below Figure. The gravel then passes off the end of the belt. The drive wheels are 80 cm in diameter and rotate clockwise at 150 r/min. Neglecting system friction and air drag, estimate the power required to drive this belt.

19. A pump in a tank of water directs a jet at 45.0 ft/s and 200 gal/min against a vane, as shown in the figure. Compute the force $F$ to hold the cart stationary if the jet follows (a) path A; or (b) path B. The tank holds 550 gallons of water at this instant.
20. Water at 20°C flows down a vertical 6-cm-diameter tube at 300 gal/min, as in the figure. The flow then turns horizontally and exits through a 90° radial duct segment 1 cm thick, as shown. If the radial outflow is uniform and steady, estimate the forces ($F_x$, $F_y$, $F_z$) required to support this system against fluid momentum changes.

21. A elbow-shaped tube in below Figure, with constant cross-sectional area $A$ and diameter $D \ll h, L$. Assume incompressible flow, neglect friction, and derive a differential equation for $dV/dt$ when the stopper is opened. Hint: Combine two control volumes, one for each leg of the tube.

22. The wye joint in below Figure splits the pipe flow into equal amounts $Q/2$, which exit, as shown, a distance $R_0$ from the axis. Neglect gravity and friction. Find an expression for the torque $T$ about the $x$ axis required to keep the system rotating at angular velocity $\Omega$.

23. The pipe bend of below Figure has $D_1=27$ cm and $D_2=13$ cm. When water at 20°C flows through the pipe at 4000 gal/min, $p_1=194$ kPa (gage). Compute the torque required at point $B$ to hold the bend stationary.
24. A liquid of density $\rho$ flows through a 90° bend as in below Figure and issues vertically from a uniformly porous section of length $L$. Neglecting weight, find a result for the support torque $M$ required at point $O$.

25. Given is steady isothermal flow of water at 20°C through the device in below Figure. Heat-transfer, gravity, and temperature effects are negligible. Known data are $D_1=9$ cm, $Q_1=220$ m$^3$/h, $p_1=50$ kPa, $D_2=7$ cm, $Q_2=100$ m$^3$/h, $p_2=225$ kPa, $D_3=4$ cm, and $p_3=265$ kPa. Compute the rate of shaft work done for this device and its direction.

26. Kerosene at 20°C flows through the pump in below Figure at 2.3 ft$^3$/s. Head losses between 1 and 2 are 8 ft, and the pump delivers 8hp to the flow. What should the mercury-manometer reading $h$ ft be?

27. A pop-up sprinkler head for a lawn watering system is illustrated in below Figure. The water flows upward through a supply line of diameter 1 in. at an average velocity of 30 ft/s. The water then moves radially outward between the two parallel solid disks. The disks are 4.0 in diameter and are separated by a gap of 0.20 in. Calculate the water velocity as it leaves the sprinkler head.