Performance of Aitken Accelerator in Dynamic Analysis by Subspace Method

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ABSTRACT: This article investigates the performance of Aitken accelerator in analysis by one-dimensional generalized subspace method. At first, it has dealt with non-linear dynamic analysis by generalized subspace technique and the necessity of extending analysis by this method. Thereafter, one-dimensional subspace procedure is presented. According to requirements of the process to iteration in each time step, the approach is modified with the help of Aitken accelerator. Finally, the numerical examples, according to modified algorithm, are presented.

KEY WORDS: One-Dimensional Generalized Subspace, Aitken Accelerator, Nonlinear Dynamic Analysis, Plates, Shell

INTRODUCTION

Analysis of particular shaped structures under different loading conditions, unusual hardening and different support conditions, especially in dynamic nonlinear behavior has some limitations, whereas finite element seems to be so practical because of its ability in dealing with all these complexities.

Since dynamic analysis in non-linear situations is incremental-iterative and in real situations, using a lot of elements for modeling the system is a necessity, so this analysis would consist of a large system of simultaneous equilibrium equations in each node which is so time consuming especially in non-linear and time dependent situations.

Whereas most of the time, number of selected freedom degree, especially in nonlinear analysis is because of complexity of the topology of system and loads and not because of the complicated expected responses [1], some techniques for decreasing number of freedom without losing appropriate accuracy of responses came into exist.

Dimension decreasing of primary system is a kind of mapping to a subspace that is accomplished with using a linear conversion like, \( Z = TX \). \( X \) is a \( n \) dimensional arbitrary vector and \( Z \) is its \( m \) dimensional portrait. \( T \) is the \( n \times m \) dimensional converter matrix.

But the problem is, mapping is always accompanied by losing information. Efforts for minimizing this error caused to occurrence of one-dimensional subspace method. In this method, by separating simultaneous dynamical equilibrium equation, instead of solving \( m \) simultaneous equilibrium equation, \( m \) separated one dimensional equilibrium equation will be solved.

Since for modifying results, an iterative solution is required, for creating a competitive method in comparison to ordinary methods, numbers of these iterations have to be minimized.

DIMENSION DECREASING

Motion equation in dynamic situation is according to equation 1:

\[
M\ddot{U} + C\dot{U} + KU = F
\]

\( M, C, K \) and \( F \) are respectively mass, damping and stiffness of system.

\( \dot{U}, \ddot{U} \) and \( F \) are respectively displacement, velocity, acceleration and load.

For using subspace method, the displacement of system \( U(t) \) with \( n \) dimension is defined as linear combination of generalized displacement \( Y(t) \) with \( n \) dimension.

\[
U = Y \Phi \Psi^T
\]

This equation shows the conversion of coordinate system. Converter matrix \( \Phi \), consists of \( m \) columnar vector \( \Psi_i \) (with \( n \) dimension) that are independent and complete. With converting velocity and acceleration the same as displacement and also multiplying \( \Psi^T \) by equation, converted system equation will be an equation 3:

\[
\bar{M}\ddot{Y} + \bar{C}\dot{Y} + \bar{K}Y = \bar{F}
\]

\( \bar{M}, \bar{C}, \bar{K} \) and \( \bar{F} \) are \( m \) dimensional square converted matrixes and \( \bar{F} \) is \( m \) dimensional converted vector.

Stages for dimension decrease consist of:

1- Creating basic vectors of subspace
2- Cessation criteria of creating basic vectors of subspace
3- Calculating generalized matrices and primary conditions vectors of consecutive time steps
4- Forming system equation in generalized subspace method
5- Solving equations and converting it to real space.

The first suggested vectors in this method were eigenvectors but as far as they are not the most suitable ones, other vectors are suggested [2]. Some of suggested vectors are Ritz-Wilson vectors, buckling mode, modal derivatives.

Used vectors in this research are Ritz vectors [4]. First stage for determining this collection of orthogonal vectors is specifying place distribution of loads. The first Ritz vector is statistical response to load vector \( F \) [4]. In the